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Behavior of Metals under Complex Static and Alternating Stresses

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BEHAVIOR OF METALS UNDER COMPLEX STATIC AND ALTERNATING STRESSES

7.1 Introduction

In this chapter we will examine fatigue-test data in order to obtain a generalization of how stresses combine to cause fatigue failure, and we will then apply the generalization to a few examples.

The local nature of the phenomenon of fatigue failure makes it necessary to trace carefully the path which the applied loads take within the structure, because a fatigue crack can start in one small region of high alternating stress and lead to complete failure regardless of how strong the rest of the structure is. The loads must be traced through the structure to the subcomponents, and the stress distribution within the subcomponents must be analyzed to find the critical regions of high stress. Typical regions of high local stress are at rivet holes, oil holes, punch marks, roots of threads, and other sharp reentrant corners.

But after the critical regions have been found and their stresses calculated, how does one know what variation of stress is permissible? The stress is usually not a simple stress, but some combination of alternating normal stress and alternating shear stress superimposed upon static stresses. It is not feasible to test every material under all combinations of combined alternating stresses and combined static stresses, but some estimate must be made from the common laboratory tests on each material.

Several criteria of how stresses combine to cause fatigue failure have been proposed, but none of them permits the engineer to take full advantage of the strength of the materials. Because it is usually desirable to minimize the inertial forces of moving parts, the weight of parts to be lifted, and the size or the cost of the mechanism, it is desirable to obtain the necessary strength without use of excessive material. The experimental results which lead to the generalization in a criterion will be presented and the more conservative criteria discussed in their light.

7.2 Description of Stress

Stresses are what we are concerned with; therefore, so that they can be discussed with precision, their description will be quickly reviewed. The stresses on any plane within a body can be broken up into their normal and tangential components.[31] The normal stress is either compressive or tensile, and the tangential stress, which is called the shear stress, has some direction on the plane. To describe the stresses in a small volume of material, enclose it in an imaginary cube and give the normal and shear stresses on the cube's six faces. The description of the stresses in the volume reduces to the stresses on three orthogonal planes if the cube is made so small that the stresses on opposite faces are equal. The theory of elasticity shows that the cube can always be oriented in such a way that there are no shear stresses on the planes, only normal stresses. These three normal stresses and their directions completely describe the stress state at a point. Thus the static stress at a point within a body, no matter how many forces are applied or how complicated is the shape of the body, can always be given by the three principal stresses and their directions. These normal stresses are called the principal stresses and are ordered $P_1 > P_2 > P_3$.

If a material is isotropic or possesses the quasi isotropy associated with the randomly oriented microscopic crystals of a polycrystalline body, the direction of the principal stress is immaterial and only the magnitude is significant.

7.3 Stress Criteria

To illustrate the use of principal-stress description and to introduce the concept of failure criteria, several static-failure criteria will be presented.

The criterion that predicts fracture for brittle materials such as glass and hardened tool steels is the maximum-tensile-stress criterion. When P_1, the greatest positive principal stress, exceeds a certain critical value for the material, fracture occurs. This simple criterion can be expressed as

$$P_1 > \sigma_{crit} \qquad \text{fracture}$$

This prediction is independent of P_2 and P_3.

A criterion which roughly predicts the onset of plastic deformation of

metals is the maximum-shear criterion, which states that flow will occur when the shear stress exceeds a certain critical value. The maximum shear stress is shown in the theory of elasticity to be equal to half the difference of the greatest and least principal stresses. This is expressed mathematically as

$$\frac{P_1 - P_3}{2} > \tau_{crit} \qquad \text{slip}$$

This criterion is independent of the intermediate stress P_2.

A criterion which more closely predicts plastic flow is the octahedral-shear criterion. The predictions by this criterion never differ from the above maximum-shear-stress criterion by more than 16 per cent. It averages the effect of shear stresses on many differently oriented slip planes.[22]

$$\tfrac{1}{3}\{(P_1 - P_2)^2 + (P_2 - P_3)^2 + (P_1 - P_3)^2\}^{1/2} > \tau_{crit\text{-}oct}$$

7.4 Fatigue Stresses

The above criteria have been very useful in predicting failure from static stresses. The stresses that result in fatigue failure present a more

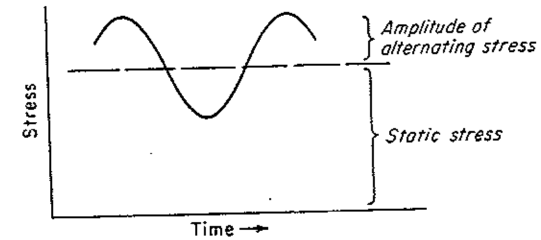


Fig. 7.1 Definition of fatigue-stress components.

difficult problem because the three principal stresses and their direction may individually fluctuate in any arbitrary, complicated way. In order to break the problem down to a size that can be analyzed, let us see how it can be simplified. In Chaps. 2 and 12 a good start is made when the actual stress spectrum is replaced by an equivalent one of cyclic stresses of constant amplitude. If the principal stress is a simple sinusoidal one, it can be split into an alternating stress superimposed upon a static stress. The alternating component can be described by its amplitude, as shown in Fig. 7.1. Let us also assume that the directions of the principal stresses remain fixed and that each principal stress varies at the same frequency. The analysis will be further simplified by limiting it to the surface and assuming that no loads are applied to the surface at the point under consideration. This is not so limiting as it first may seem, because most fatigue cracks begin at the surface of machine parts where at most a biaxial state of stress exists. These cracks start at the surface instead of down inside because:

1. Many parts have bending or twisting moments applied to them, thus causing the highest stresses to be at the surface.

2. Surface stresses may be further increased by the presence of stress concentrations such as scratches, roughness, tool marks, and notches.

3. Some investigators of fatigue feel that the crystalline grains at the surface are inherently weaker because their deformation is not confined by neighboring grains on the one side.

The criteria for static failure which we have just examined naturally contain only the static stresses, but any fatigue criterion must contain the alternating stresses which cause the fatigue failure, and the superimposed static stresses which can influence the failure.

7.5 Combined Alternating Stresses

First, experiments on the behavior of materials under various combinations of alternating biaxial stress will be examined. After this behavior has been established and generalized, other experiments will be studied to see what effect static stresses have when they are superimposed upon the alternating. Experiments by H. J. Gough[5] on different combinations of alternating bending and torsion will be examined, for they are extensive and agree with the results of others.[4,20]

A convenient and very general way to present the results of combined-stress fatigue tests is to plot the amplitudes of the alternating principal stresses which caused failure for a certain number of repetitions of combined stress. The abscissa of Fig. 7.2 is the amplitude of one principal stress, and the ordinate is the other. The axes are labeled σ_1 and σ_2, and these are the principal stresses when they are properly ordered $P_1 > P_2 > P_3$. In the lower quadrant, σ_2 is the greatest principal stress P_1, and the least, P_3, is the negative σ_1. The intermediate principal stress P_2 is zero because the region under consideration is at the free surface to which no stresses are assumed to be applied and its principal direction is normal to the surface. Combined bending and torsion data appear only in the lower quadrant. This can be seen by noting that, in pure torsion (pure shear), the principal stresses are equal in magnitude and opposite in sign. This stress state is represented by points along line 0-2. As more bending is applied in the 2 direction, the stress state is represented by points closer to the σ_2 axis. Pure bending in the 2 direction is represented by points on the σ_2 axis.

It must be remembered that the stresses alternate during each cycle, going from a given tensile stress to an equal compressive stress on the other half cycle. The lower quadrant represents the case where the principal stresses are out of phase; that is, at the time one stress reaches its greatest tensile value, the other principal stress reaches its greatest

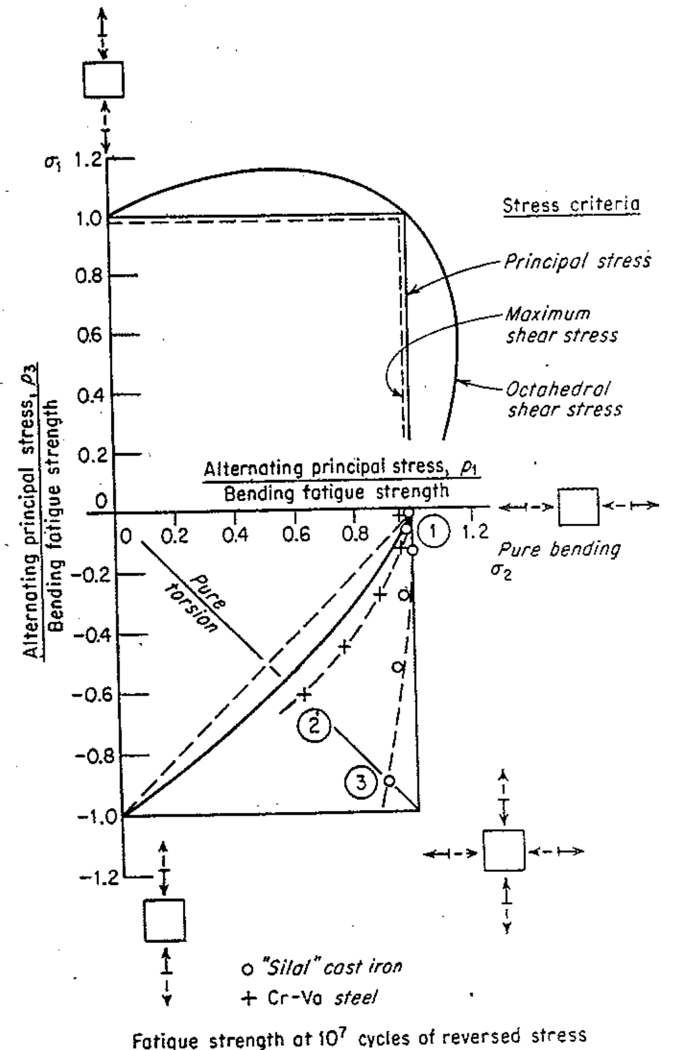


Fig. 7.2 Comparison of Gough's combined bending and torsion data[5] with failure criteria.

compressive value. In the upper quadrant the stresses are in phase. Here both stresses become tensile at the same time and then become compressive together in the opposite half cycle.

The test data are plotted for a given cyclic life, say 10^7 cycles, for different combinations of alternating stress for the particular material under consideration. The line connecting these test data separates the σ_1-σ_2 plane into two regions. Every stress combination represented by a point within the region can be safely applied 10^7 times to the material without failure; all points outside this region will result in failure before this number of cycles.

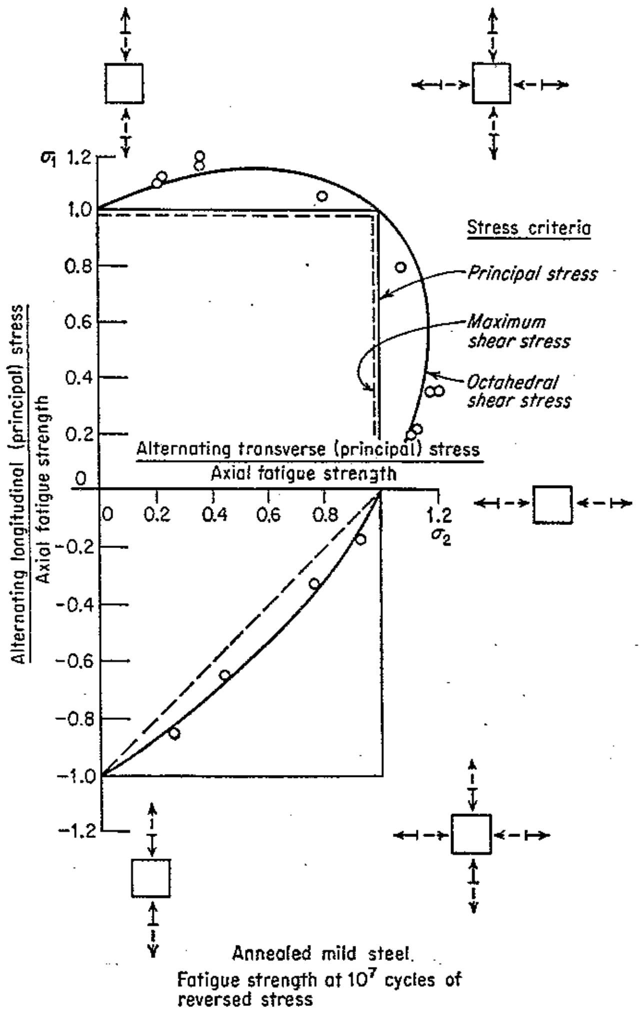


Fig. 7.3 Comparison of Sawert's data[23] with failure criteria.

Gough's test data for two materials are plotted in Fig. 7.2. The coordinates are reduced to nondimensional form by dividing each by the reversed-bending fatigue strength. The curve 1-2 for the chromium-vanadium steel is typical of one group of material which consists of most steels in most conditions, aluminum, bronze, etc., while the other curve, 1-3, is typical of cast irons and hardened hypereutectoid steels. The

marked difference in behavior between these two groups of materials will be discussed later.

Obtaining data for the lower quadrant by conducting combined bending and torsion tests is relatively easy compared to obtaining data for the upper quadrant. The experimental difficulties were overcome and data obtained for both quadrants by Sawert.[23] He applied an alternating load to a series of differently shaped specimens, each shape having been calculated to develop the desired combination of stresses. His results for an annealed low-carbon steel are shown in Fig. 7.3. His tests on chromium-vanadium steel are similar to those on low-carbon steel presented here.

The static-failure criteria which were discussed previously are superimposed upon Gough's and Sawert's fatigue-test data. In both the upper and lower quadrants the data agree reasonably well with the octahedral-shear criterion. The maximum-normal-stress criterion is not far removed from the data in the upper quadrant, but it is far removed in the lower one. There are, however, two difficulties with the data. As is usual in fatigue testing, it is impractical to break enough specimens to obtain a good average, and in Sawert's work we do not know the extent of "size effect"; his specimens, because of the combined stresses he wanted, were necessarily of different sizes and shapes. "Size effect," which is discussed in detail in Chap. 13, is a description of the behavior of a material which in a large part appears to have a lower strength than when it is formed into a smaller part. Nevertheless, the data for steel do seem to agree fairly well with the octahedral-shear-stress criterion. Other tests of Gough's, which are not shown here, center about this criterion.

Another indication of the stress criterion can be seen by examining the ratio of reversed-torsional fatigue strength to the reversed-bending fatigue strength for Gough's tests presented here:

Material	*Max Shear Stress in Torsion Test* / *Max Shear Stress in Bending Test*
0.1% C steel, normalized	1.13
0.4% C steel, normalized	1.25
0.4% C steel, spheroidized	1.135
0.9% C steel, pearlitic	1.37
3% Ni steel	1.20
3–3½% Ni steel	1.20
Cr-Va steel	1.20
3½% Ni-Cr steel, normal impact	1.305
3½% Ni-Cr steel, low impact	1.27
Ni-Cr-Mo steel, 60–70 ton	1.08–1.17
Ni-Cr-Mo steel, 75–80 ton	1.04
Ni-Cr steel, 95–105 ton	1.175
Silal cast iron	1.82
Nicrosilal cast iron	1.67

The points representing the two stress states on the principal-stress diagram (Fig. 7.2) are:

1. Pure bending stress, located on the abscissa
2. Pure torsional stress, located on the line passing through the origin and bisecting the lower quadrant

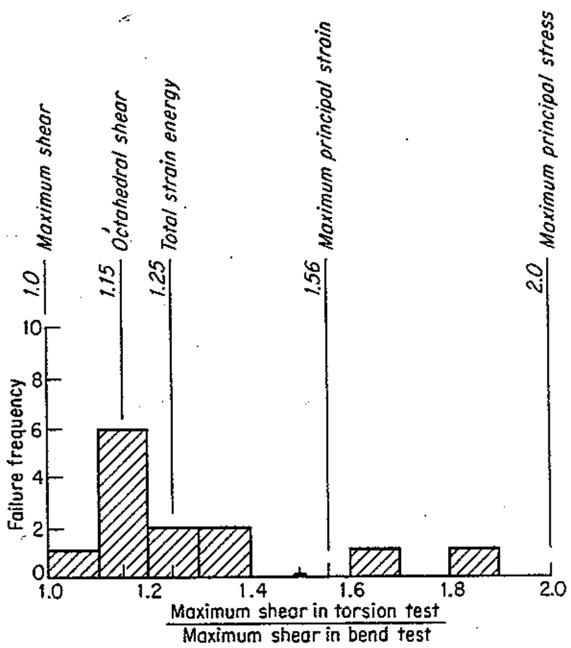


Fig. 7.4 Comparison of Gough's torsion and bending fatigue tests[5] with failure criteria.

The ratios of the maximum shear stress in the torsion test for failure to the maximum shear stress in the bending test for failure are:

Criterion of Failure	*Max Shear Stress in Torsion Test / Max Shear Stress in Bending Test*
Constant maximum shear stress	1.00
Constant maximum octahedral shear	1.15
Constant total strain energy	1.25
Constant maximum principal strain	1.56
Constant maximum principal stress	2.0

The ratios from this table are plotted in Fig. 7.4 in frequency of occurrence in 0.1 intervals. The ratio for one pair of tests falls between 1.0 and 1.1, and six ratios fall between 1.1 and 1.2, two between 1.2 and 1.3, and two between 1.3 and 1.4. The two ratios above this were for tests on cast irons which will be considered separately. The ratio for the octahedral-shear criterion is 1.15, the middle of the interval in which most of the test ratios fall.

Notice that the maximum-shear-stress criterion also agrees fairly well with data presented in Figs. 7.2, 7.3, and 7.4. Constants can be so chosen that these two criteria do not differ by more than ±8 per cent. The physical significance of the two criteria is much the same. The octahedral-shear-stress criterion is an expression of the average of shear on different directions of all crystals in the aggregate, with the slip on any given plane being caused by a component of the shear stress in the slip direction.[22] The deviations from the octahedral criterion can indicate the incompleteness of the averaging.

Postulating that the potential energy of elastic distortion must not exceed a critical value is mathematically the same as the octahedral-shear-stress criterion, but this interpretation does not seem to the author to have as much physical significance as that of the averaging of the shear stresses on different planes.

7.6 · Anomalous Behavior of Cast Iron

The data for cast iron presented in Fig. 7.2 and the ratios of torsional to bending fatigue strengths for cast iron in Fig. 7.4 fall nearer to the maximum-principal-stress criterion than to the shear criteria. Let us examine the nature of cast iron to see if there is some reason for this difference in behavior. Microscopic observation of cast iron shows many flakes of graphite dispersed throughout the metal. Graphite has a modulus of elasticity much lower than that of the surrounding iron crystals. Therefore, the flakes of graphite carry very little stress in compression, and because of their low strength they carry even less in tension. They act like holes in the iron structure. The effect of these flakes can be seen by considering a simplified model of three holes oriented at different angles as shown in Fig. 7.5.

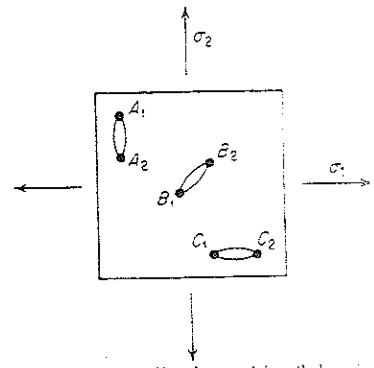


Fig. 7.5 Idealized graphite flakes in cast iron.

When σ_1 is applied, the regions A_1 and A_2 at the edge of the one flake will be much more highly stressed than any other region because of the elastic-stress concentration caused by the hole. Regions B_1 and B_2 will also be more highly stressed than the average, but still considerably less than A_1 and A_2. The regions C_1 and C_2 around the edge of the flake longitudinal to the applied stress will be stressed even less than the average for the bulk of the material. The regions most highly stressed by the transverse stress σ_2 are C_1 and C_2, while regions A_1 and A_2 are stressed very little. The detailed analysis of cracks oriented at different angles under several combinations of stress shows that the sum of the effects caused by the transverse and longitudinal stresses together at cracks oriented at angles between that of A_1-A_2 and C_1-C_2 is less than that caused by them individually at A_1 and A_2 or C_1 and C_2 except for the range of combinations of applied stresses where the transverse stress approaches a value equal to the longitudinal one. In that case the combined effect is only slightly greater at one orientation than the isolated individual effects are at other orientations.[24] Thus the greatest damage from the two stresses is localized at different points, which causes their effects to be independent. The independence of the effects of the principal stresses is the essential characteristic of the behavior that indicated that the principal-stress criterion might be operating; however, the independence is explained by the localization of the damage.

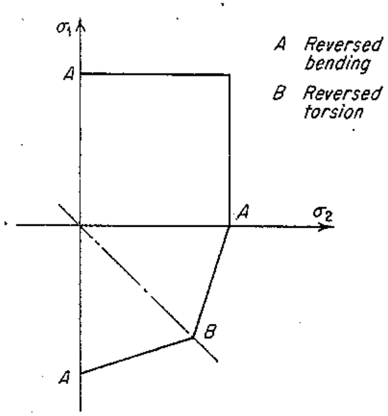


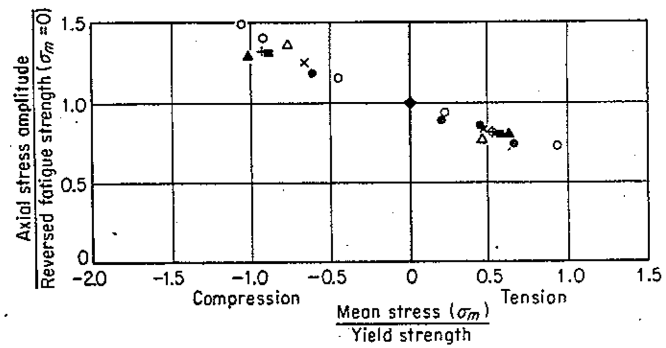
Fig. 7.6 Failure of cast iron under combined stresses.

In all cases examined in which a maximum-normal-stress criterion best fitted the data, there has been strong reason to believe that flakes of graphite or other flakelike inclusions were present in the material. Turner's tests on annealed hypereutectoid steel[32] show that a maximum-normal-stress criterion was more nearly satisfied than a maximum-shear-stress criterion. Graphite flakes may well have been present in the steel he used, for Sisco[25] mentions that "many years ago it was noticed that occasionally in annealing high-carbon steel, free carbon (graphite) would form." The presence of only a few flakes might cause the behavior to occur.

Material with flakelike soft inclusions or cracks will be excluded from detailed consideration here; however, a useful criterion is shown in Fig. 7.6 and is in accordance with the ideas of Guest[6] and Stulen and Cummings.[29] The criterion is identical to the principal-stress criterion in the upper quadrant. In the lower quadrant it differs from the principal-stress criterion by having a straight line A-B connecting the point representing the fatigue strength in bending to that for torsion (Fig. 7.6).

7.7 Simple Combinations of Alternating and Static Stresses

From the examination of the above data on the effect of different combinations of purely alternating stress it appears that the alternation of shear stress causes the fatigue damage. Most machine parts have static stresses superimposed upon the alternating ones, and their combined



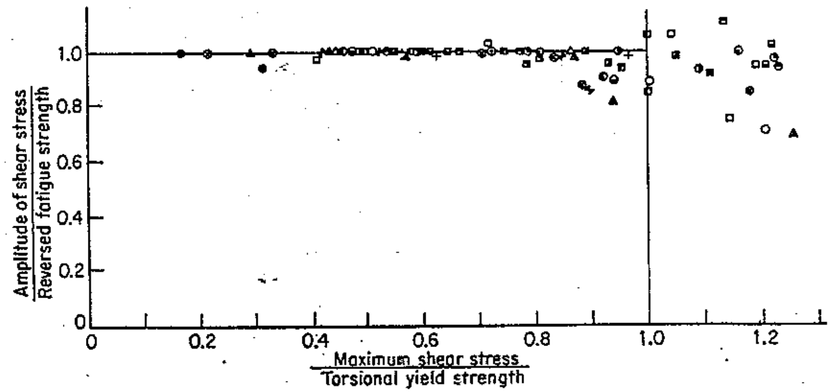
Material	Fatigue strength, psi	Yield, psi	Source
× 24 ST aluminum	± 26,000	48,000	Newmark [16]
o 0.41C steel	± 36,000	55,000	Nishihara [18]
+ 0.65C steel	± 38,000	57,000	Nishihara "
△ 0.44C steel	± 33,000	57,000	Nishihara "
• Duralumin	± 17,000	32,000	Nishihara [17]
■ Mild steel	± 26,000	38,000	Roš [21]
▲ Mild steel	± 37,000	47,000	Roš "

Fig. 7.7 Effect of mean axial stress on axial fatigue.

effect must be determined. Experiments on different simple combinations of static and alternating stress will now be examined. They are:

1. The effect of static tension on the permissible amplitude of alternating axial stress (Fig. 7.7)
2. Static compression on the permissible amplitude of axial alternating stress (Fig. 7.7)
3. Static torsion on the permissible amplitude of alternating torsion (Fig. 7.8)
4. Static torsion on the permissible amplitude of alternating bending (Fig. 7.9)
5. Static tension on the permissible amplitude of alternating torsion
6. Bending on the permissible amplitude of alternating torsion
7. Compression on the permissible amplitude of alternating torsion (Fig. 7.10)

It is common engineering practice to present the data on the effect of static stress on fatigue by plotting it on the Goodman diagram.[1] This is impossible when the static stress is not applied in the same direction as the alternating stress; therefore, for uniformity, all of the data will be



Material	Source
+ Cr-Ni steel, quenched tempered	P. Ludwik and J. Krystok
● Siemens-Martin steel	
● Baustahl, soft	
▫ Schmieder bronze A, Rolled	
△ Aluminum alloy 17ST	Aluminum Company of America
▲ Aluminum alloy 27ST	
△ Aluminum alloy 53ST	
▪ Mild steel, hot rolled	G. A. Hankins
▪ 0.6% C steel, quenched tempered	
o Si-Mn steel, quenched tempered	
◉ Cr-Va steel, quenched tempered	
◉ Brass, 60 Cu-40 Zn as rolled	H. F. Moore and R. E. Lewis
◉ Copper, commercial pure, cold rolled	
◉ Duralumin, as rolled	
◉ Malleable iron, untempered	A. Pomp and M. Hempel
◉ Malleable iron, tempered	
● Beryllium bronze, 97.6% Cu-2.4% Be	J. B. Johnson
+ 0.9% C steel, quenched tempered	
+ Cr-Va steel, quenched tempered	
▫ SAE 3140 steel, quenched tempered	J. O. Smith
▫ SAE 3140 steel, as hot rolled	
▫ Tobin bronze, cold rolled	
▫ 1.2% C steel, normalized	H. F. Moore and T. M. Jasper
▫ 3.5% Ni steel, special treatment "A"	
▫ 3.5% Ni steel, special treatment "D"	
▫ 0.49% C steel, normalized	
▫ 0.46% C steel, quenched drawn	D. J. McAdam Jr.

Fig. 7.8 Effect of static stress on range of shear stress.[26]

presented by plotting the amplitude of the alternating stress on the y axis and the static stress on the x axis.

Figure 7.7 shows the effect of mean axial stress on the permissible amplitude of alternating axial stress for some common materials. The number of cycles to failure was between 10^6 and 10^7 for this group of tests. The tests show clearly that the amplitude of alternating stress is decreased

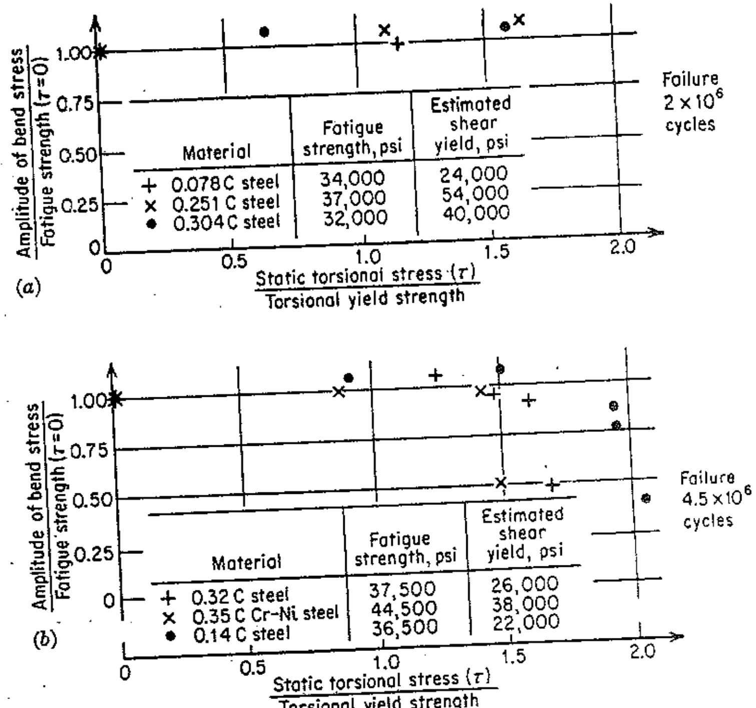
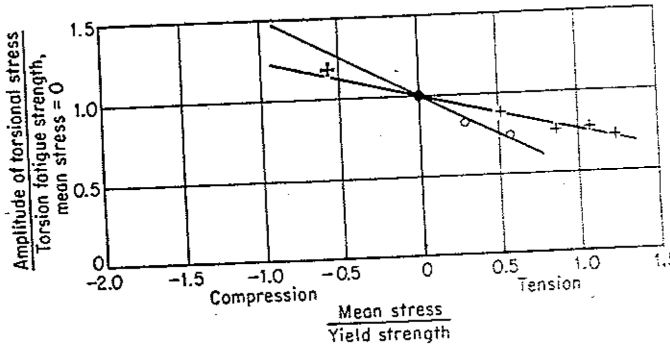


Fig. 7.9 Effect of static shear on permissible amplitude of alternating bending stress. (*a* is from Ref. 19; *b* is from Ref. 9.)



Material	Mean stress	Fatigue strength, psi	Yield, psi	Source
o Ni-Cr-Mo steel	Bending	±53,800	137,000	Gough [5]
+ Mild steel	Axial	±16,400	33,000	Hohenemser and Prager [8]
✢ 61S-T aluminum	Axial	±13,000	40,000	Sines [24]

Fig. 7.10 Effect of static normal stress on torsional fatigue.

by tensile mean stress and is increased by compressive mean stress. The mean-stress coordinate was made nondimensional by dividing it by the yield strength. This was done not to imply a dependence upon the yield strength, but merely as a convenient way of presenting the data for different materials on the same figure.

The data for Fig. 7.7 were critically selected; if all other investigations were included, the trend would be horizontal.[26] It is difficult to apply a true axial compressive load, and it is likely that uncontrolled bending stresses have invalidated some tests. The tests presented here were chosen because the testing methods used ensured true axial loading. In Nishihara's tests, the load was transmitted to the specimen through an arrangement containing a ball resting on a flat surface. Thus the exact point of application of the force was known, and it could be positioned on the exact axis of the specimen. Newmark and Roš used specimens with carefully squared ends in attachments that made the gripping heads remain parallel and concentric during the stress cycle. They also checked the axiality by means of electric strain gages.

The relation appears to be linear in the region where maximum stress does not exceed the yield strength. If only fatigue rupture (not yielding) is considered to be a failure, the linear part of the curve has been shown to extend beyond the yield strength and then turn sharply to the abscissa, intersecting it at the point of static-rupture strength.[21]

Figure 7.8 shows the effect of static torsion on the permissible amplitude of alternating torsion. The great number of tests plotted gives conclusive proof that static torsion has no effect on the permissible range of alternation as long as the maximum torsional stress is below the yield strength.

Figure 7.9 shows the available data on the effect of static torsion superimposed on alternating bending stresses. From the few tests plotted, it seems that bending-fatigue strength is not affected until static torsion exceeds the torsional yield strength by almost 50 per cent.

Figure 7.10 shows only three tests on the effects of axial static stress on the permissible amplitude of alternating torsion, but they are the only ones in print known to the author. The linear dependence of the amplitude of alternating torsion upon the applied static axial stress is similar to that shown in Fig. 7.7, which presents the effect of static axial stress on the amplitude of axial stress.

7.8 Analysis of Combined-stress Fatigue Data

The results of the tests under simple combinations of static stress with alternating stress are summarized in Fig. 7.11. Because the alternation of shear stress seems to cause fatigue failure, the static stresses on the planes of greatest alternation of shear stress will be examined to see if any

correlation can be made between them and the effect on the permissible alternating stress. The planes are identified on the specimens and are shown magnified in the center column with the static stresses which act on them.

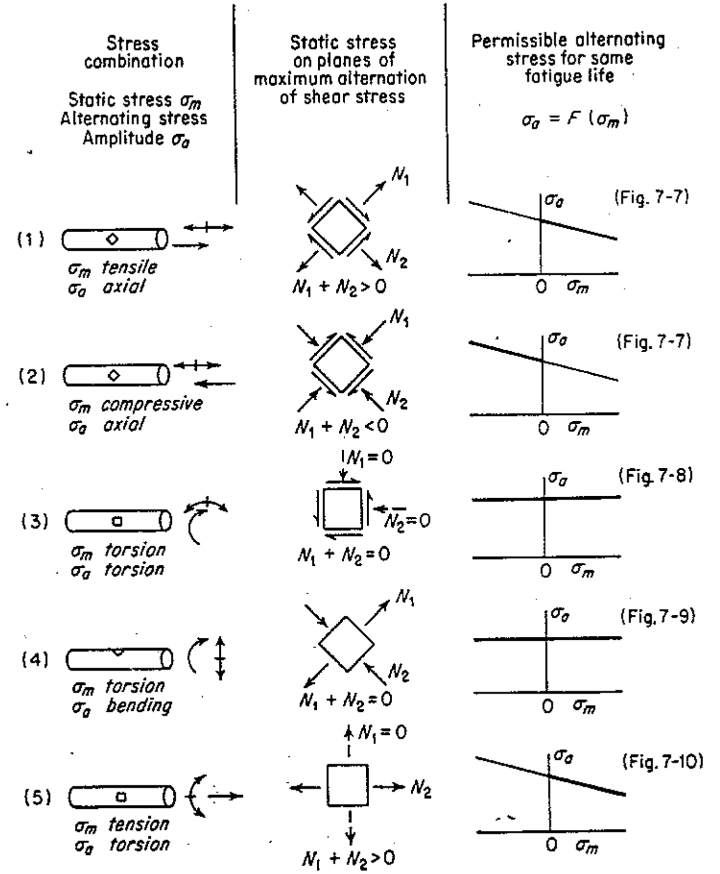


Fig. 7.11 Summary of the effect of different combinations of static and alternating stresses on fatigue life.

Let us look at the sum of the normal stresses which act on these planes. When the sum $N_1 + N_2$ is positive, as it is for cases 1 and 5, an increase in the static stress reduces the permissible alternation of stress. When the sum is negative, however, as in case 2 the permissible alternation is increased. And when the sum of $N_1 + N_2$ is always zero regardless of the applied static stress, as in cases 3 and 4, the static stress has no effect. Thus there appears a simple correlation between the alternation of stress and the sum of N_1 and N_2.

Since $N_1 + N_2$ is shown to be independent of the directions 1 and 2 as long as they are on planes at right angles to each other,[31,p.187] it really was not necessary to have examined the planes of greatest alternation of shear, but the correlation could have been seen from the sum of static normal stresses on any two orthogonal planes.

7.9 Criterion for Fatigue Failure

Looking back over the data of Figs. 7.4 to 7.7, it seems that the relation which exists between the effect of the static stress and the permissible amplitude of alternating stress is linear, or at least the accuracy of the test data does not justify higher-order terms. A criterion can now be proposed which includes the effect of different combinations of alternating stress with static stresses. It is the simple statement that the permissible alternation of the octahedral-shear stress is a linear function of the sum of the orthogonal normal static stresses. It is mathematically expressed as

$$\tfrac{1}{3}\{(p_1 - p_2)^2 + (p_2 - p_3)^2 + (p_1 - p_2)^2\}^{1/2} \leq A - \alpha(S_x + S_y + S_z)$$

where p_1, p_2, and p_3 are the amplitudes of the alternating principal stresses and S_x, S_y, and S_z are the orthogonal static stresses. The A is a constant for the material, proportional to the reversed fatigue strength, and α gives the variation of the permissible range of stress with static stress. Both A and α are given for the desired cyclic lifetime. The expression on the left must not exceed the right-hand side, or failure will occur before the desired lifetime. The amplitudes of the alternating principal stresses do not have to be ordered $p_1 > p_2 > p_3$ in the criterion because their differences are squared in it.

There are recent test data by Findley[3] which show that for some materials the dependence of the permissible alternation of stress upon the static stress is not great; that is, the constant α of the criterion may be near zero. However, this was not true for test data presented here.

7.10 Determination of Constants for Criterion

The values A and α which describe the fatigue properties for a material can be determined from two fatigue curves in which the static stresses are appreciably different. Two curves which are convenient for their determination are the reversed axial test and the zero-tension fluctuating stress. To illustrate the simplicity, the calculation of A and α will be made from these two determinations of the fatigue properties.

In the reversed axial test the criterion

$$\tfrac{1}{3}\{(p_1 - p_2)^2 + (p_2 - p_3)^2 + (p_1 - p_3)^2\}^{1/2} \leq A - \alpha(S_x + S_y + S_z)$$

reduces to

$$\frac{\sqrt{2}}{3} p_1 = A$$

because S_x, S_y, S_z, p_2, and p_3 are all zero. Thus $A = (\sqrt{2}/3)f_1$, where f_1 is the amplitude of the reversed axial stress, which would cause failure at the desired cyclic lifetime.

For the zero-tension fluctuating-stress cycle, the criterion reduces to

$$\frac{\sqrt{2}}{3} p_1' = A - \alpha p_1'$$

because p_2', p_3', S_y', and S_z' are zero and $S_x' = p_1'$. Solving the above equation,

$$\alpha = \frac{A}{p_1'} - \frac{\sqrt{2}}{3} = \frac{\sqrt{2}}{3}\frac{f_1}{f_1'} - \frac{\sqrt{2}}{3}$$

where f_1' is the amplitude of the fluctuating stress which would cause failure at the same lifetime as the reversed stress f_1.

7.11 The Graphical Presentation of the Criterion

For a free surface, the criterion is graphically presented in Fig. 7.12. At a free surface the one principal stress is zero, thus permitting it to be plotted as a two-dimensional function of the other two principal stresses. It appears as a series of "concentric" ellipses, the size of the ellipse depending on the sum of the static normal stresses. The more positive (tensile) the sum of the static stresses, the smaller the ellipse; conversely the more negative the sum, the larger the ellipse.

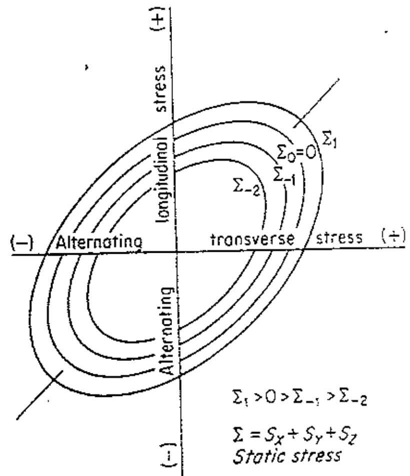


Fig. 7.12 The plotted stress criterion.

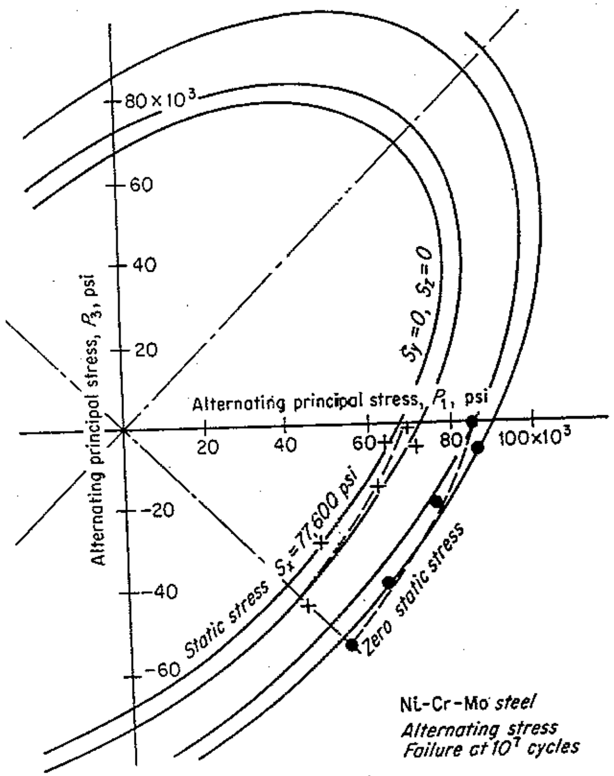
As shown in Sec. 7.10, A and α can be found for the desired cyclic lifetime and the criterion plotted as shown in Fig. 7.12. If the static stresses are fixed, the permissible combinations of alternating stress amplitudes can be chosen. The ellipse corresponding to the sum of the static stress is selected, and any combination of alternating stresses within the area it encloses is safe; conversely, all combinations on the outside will cause premature failure.

If the criterion is to be applied to the interior of a body where all of the three principal stresses are different from zero, the graphical representation becomes a three-dimensional plot with the third principal stress orthogonal to the other two. The failure surface in this space is a cylindrical surface with generators having the directional cosines $(1/\sqrt{3},$

$1/\sqrt{3}$, $1/\sqrt{3}$) intersecting the planes of the axes in the ellipses shown in the two-dimensional representation.[7]

7.12 Comparison of Criterion with Test Results

Very few data exist to check the criterion on more complex combinations of alternating and static stresses. One series of tests was performed



General criterion

$$\tfrac{1}{3}\{(p_1 - p_2)^2 + (p_1 - p_3)^2 + (p_2 - p_3)^2\}^{\frac{1}{2}}$$
(Alternating)
$$\leq A - \alpha(S_x + S_y + S_z)$$
(Static)

Fig. 7.13 Comparison of Gough's combined stress fatigue data[5] with general stress criterion.

by Gough on Ni-Cr-Mo steel specimens by subjecting them to different combinations of alternating bending and torsion with and without the superposition of a fixed static bending stress. Figure 7.13 shows that the test data fall close to the constant-static-stress ellipses of the criterion. Professor M. Roš, at the Swiss Federal Material Testing and Research Institute, Zurich, has conducted extensive tests on several materials

under combined stresses that varied between no stress and the maximum stresses. The specimens were hollow cylinders in which hydraulic pressure caused a hoop stress, upon which was superimposed a longitudinal stress from an external force. Some typical results are plotted in Figs. 7.14 and 7.15. Other investigators have found similar results.[10,12,14]

The stress components in these tests varied between no stress and the maximum stress; thus, the static-stress components are different for each combination of the alternating stress, and are equal to the amplitudes of the alternating stresses. Since the static stress was not held constant,

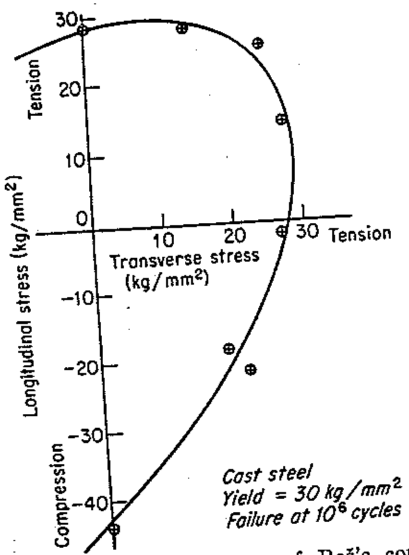


Fig. 7.14 Comparisons of Roš's combined stress fatigue data[21] with general stress criterion transformed for fluctuating cycle; minimum stress = 0.

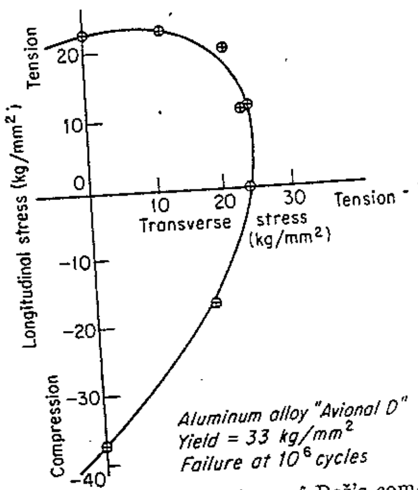


Fig. 7.15 Comparison of Roš's combined stress fatigue data[21] with general stress criterion transformed for fluctuating cycle; minimum stress = 0.

the data cannot fit the concentric ellipses of Figs. 7.12 and 7.13. However, mathematically transforming the criterion[24] for this special case by placing $S_x = p_1$, $S_y = p_2$, it plots on Figs. 7.14 and 7.15 as an ellipse inclined at 45° to the axes and displaced toward the bottom left.

It is seen that the ellipse rather closely fits the test data for the cast steel and aluminum.

7.13 Designing against Fatigue Failure

In order to check the design of a machine part to see if it will withstand the service loads, these loads must be reduced to some equivalent cyclic stress by the means discussed in Chaps. 2 and 12. The critical regions must be identified and checked to see if the equivalent cyclic stress will initiate fatigue failures. The regions of danger are those which undergo high alternation of stress or have a high static tensile stress superimposed upon an alternating stress. The regions needing careful checking can be identified either by a detailed stress analysis or by a common-sense evaluation of the regions where stress is most likely to be high. After the regions have been identified, the local stresses must be carefully determined by either mathematical or experimental stress analysis and then compared with the criterion to see if failure is predicted.

No design which is at all conservative will permit yielding. The avoidance of yielding is essential if the criterion developed here is to be applied, because in some cases (Figs. 7.8 and 7.9) the permissible alternating stress is lowered almost catastrophically when the maximum stress exceeds the yield strength.

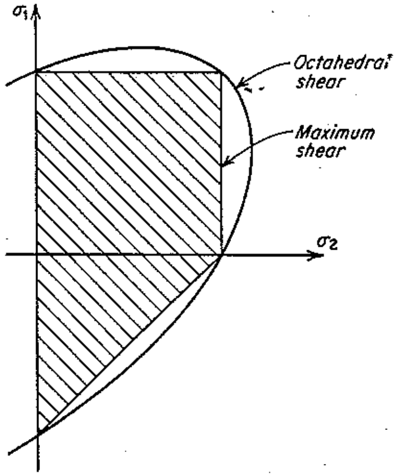


Fig. 7.16 Comparison of octahedral-shear and maximum-shear-stress criteria.

In checking a design it may be desirable to apply a more conservative criterion first if it is mathematically simpler; and, of course, if the simpler criterion is satisfied, there is no need to apply more refined criteria. A simple conservative criterion is that no local-shear-stress amplitude shall exceed one-half of the bending-stress amplitude of the standard rotating-beam fatigue strength. If static tensile stress is present at the region, the permissible amplitude should be reduced by 30 per cent. The use of rotating-beam test data, which are much more easily obtained than the axial-stress fatigue data, tends to be slightly nonconservative; however, the differences between the results of the two types of test have been shown to differ by little. A detailed study[13] sponsored by the American Society for Testing Materials showed the difference to be less than 12 per cent in the worst case and usually only a few per cent.

The 30 per cent reduction should provide for the greatest effect of static tensile stress known to the author in cases where the maximum stress does not exceed the yield strength. However, there may be reason to reduce it still further if a high biaxial static tensile stress is present.

A simple conservative criterion for avoiding yielding is that the maximum local shear stress must not exceed one-half the yield strength of the material. The simple maximum-shear-stress criterion

$$\frac{P_1 - P_3}{2} < \tau_{max} = \frac{\text{YS}}{2}$$

where the Ps are the extreme values of the principal stresses, can be seen in Fig. 7.16 to be more conservative than the precise octahedral-shear stress (Von Mises) criterion

$$\tfrac{1}{3}\{(P_1 - P_2)^2 + (P_1 - P_3)^2 + (P_2 - P_3)^2\}^{1/2} \leq \tau_{oct}$$

The region of nonyielding stress combinations predicted by the maximum-shear-stress criterion is entirely contained within the elliptical region of the octahedral-shear criterion.

7.14 An Example of Necessity of Consideration of Local Stresses

The following illustration shows the necessity of careful consideration of local stresses. In Fig. 7.8 it was seen that the permissible alternation of torque on a circular shaft was not influenced by the steady torque

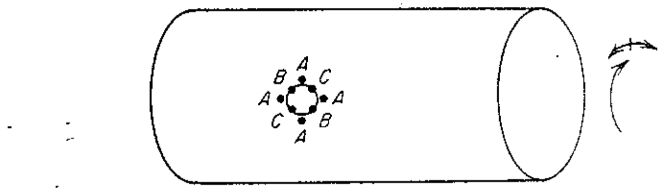


Fig. 7.17 Shaft with small transverse hole subjected to static and alternating torques.

unless the yield strength of the material was exceeded. Now let us consider the case where design requires a small transverse hole in the shaft. The superficial reaction would be that the permissible alternation of torque would still be independent of the static torque and that it would only be necessary to reduce the alternating torque to compensate for the strength reduction of the hole. Because of the local nature of the fatigue failure, that reaction would be very much in error.

Neuber[15] has calculated the stress distribution around the hole when it is small compared to the shaft. In Fig. 7.17 points of high alternation of shear stress are at A, which is a short distance removed from the edge of the hole. The sum of the orthogonal normal static stresses $(S_x + S_y + S_z)$ at A is zero, because at that point $S_x = -S_y$ and S_z, the stress normal to the free surface, is zero.

A high tensile stress, tangential to the hole, exists at point B, and a compressive one exists at C. The other principal stresses at this point on the edge of the hole are zero because the inside of the hole and the shaft surface are free surfaces. In effect, the torsional shear stress, which consists of equal orthogonal tensile and compressive stresses at 45° to the axis, is split by the hole so that two sides of the hole at 45° are in pure compression while at 90° away there is pure tension. The alternating shear stress at point B is $(P_1' - P_3'/2) = \sigma_B'/2$. The detailed analysis shows that this local alternation of shear stress is greater than that which occurs at A. Not only is the alternation of shear stress the greatest at B, but superimposed upon it is the static tensile stress σ_B, which, according to the criterion, reduces the permissible alternation of local shear stress. Thus from the detailed analysis of the local stresses inserted into the general criterion we predict that the permissible alternation of torque on a shaft with a transverse hole will be reduced by a static torque. This is indeed true for the experimental results summarized by J. O. Smith.[26]

7.15 Consideration of Factors Influencing Fatigue Strength

Factors which influence the fatigue strength of a machine part must be considered in design. These are discussed in Chaps. 2, 8, 10, and 13; however, it seems desirable to mention a few of them here.

The effect of one of the important factors, residual stress, can be predicted by the stress criterion. Macroscopic residual stresses do not dicted by the stress criterion. Macroscopic residual stresses do not differ in their effect from the static stresses caused by the loads; they should, therefore, be treated just as the static stresses are by adding them to the static stresses in the criterion. Or if it is more convenient, they can be added separately.

$$\tfrac{1}{3}\{(p_1 - p_2)^2 + (p_2 - p_3)^2 + (p_1 - p_3)^2\}^{1/2} \le A - \alpha(S_x + S_y + S_z + R_{x'} + R_{y'} + R_{z'})$$

The orthogonal axes x', y', z' for the residual stresses $R_{x'}$, $R_{y'}$, $R_{z'}$ need not be in the same directions as those for the static normal stresses.

Thus it can be seen that inducing compressive residual stresses (which are negative in sign) will permit greater alternation of stress for the same cyclic life. Conversely, tensile residual stresses are to be avoided.

Residual stresses may be intentionally or unintentionally introduced by the manufacturing processes or by the service loads. While the experimental or mathematical determination of their magnitude is not a simple matter, they can be tackled by some methods discussed in Chap. 8.

Factors, other than compressive residual stress, which may improve the fatigue strength of a critically stressed region over that predicted by the standard fatigue tests are the "size effect" and changes in the properties of the surface material. The size effect is the phenomenon that the fatigue strength of a given volume of a material under stress may be lower than that of a smaller volume of the same material. The critical region is often smaller than the standard fatigue-test specimen, e.g., the region around a no. 60 drill hole. Here the full weakening effect of the stress concentration of the hole might be almost compensated by

perhaps a twofold increase in the fatigue strength of such a small region (Chap. 13).

If the critical volume is larger than that of the common fatigue specimen, there may be some reduction of strength, but at least one test[2,p.158] shows that this reduction is not large (maximum effect here was 20 per cent).

Some processes, such as shot peening, surface rolling, and casehardening, may improve the properties of the material as well as induce residual stress. Processes may also weaken the surface. For instance, decarburization of a steel surface may occur during heat treatment and considerably reduce the fatigue strength of the steel. Of course we must remember that the fatigue fracture is not the only manner in which machines may fail. Brittle fracture from impact, excessive plastic deformation from creep, and other types of failure are important.

The many factors influencing fatigue strength, such as corrosion, temperature, and surface finish, are discussed in this book and in others.[1,2] Careful attention must be given to those processes which might reduce the strength; if they cannot be avoided, sufficient allowance must be made in the design to compensate for the possible weakening.

Criteria for fatigue from combined stresses are reviewed by J. Marin in his book "Engineering Materials."[11] In these criteria the full strength of the material is not recognized because they lead to the prediction that the static compressive stress is as deleterious as static tensile stress, while the data presented in Figs. 7.7 and 7.10 show that static compressive stress can increase the permissible alternation of stress. The criteria also suggest that static torsion would reduce the permissible range of alternating torsion, while the data given in Figs. 7.8 and 7.9 show that it has no effect unless the yield strength is exceeded.

The design procedures presented here are not in general use, but the author hopes they will serve as a useful guide where the utmost strength of the material must be utilized. Any part design which differs greatly from standard practice should be carefully fatigue-tested before it is adopted for service.

Analyses such as these try to avoid "blind safety factors," which often are multiplicative and are so often extremely wasteful. The aim is to make the parts as uniformly strong as possible. If this is done, whatever "safety factor" is applied is a true measure of safety and may be selected according to the danger either to personnel or property or of delay if failure should occur in service. The safety factor should not be used to compensate for ignorance of stress distribution, material properties, or the behavior of materials under combinations of stresses, although some allowance may be needed for the lack of information concerning the actual service loads.

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